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Observations on Genesis

"Solving a problem for which you know the answer is like climbing a mountain with a guide, along a trail someone else has laid. [But] in mathematics, the truth is somewhere out there in a place no one knows, beyond all the beaten paths. And it's not always at the top of a mountain. It might be in a crack on the smoothest cliff or somewhere deep in the valley."
—The Housekeeper and the Professor, Yoko Ogawa

The greatest conundrum of existence is that we define our reality—our *realness*—on what never can be. For us, perception is relational: velocity is fixed only by a change of position, emotion quantified only by a standard deviation from the normal[1]. But the essence of who we are—the mean of the distribution, the eye of the storm—that cannot be quantified by distance.

A system of equations requires two components to yield a singular answer: variables and constants. Centuries of study and brilliance have gone into the understanding of the variables, from the derivative to the topological group to everything in between. Variables are wild beasts that mathematicians try to tame through analysis and phase diagrams, rats in mazes that we cannot catch. And yet, the constants are the unsung heroes—without them, the system would have no solution,[2] leaving the untamed creatures to run the Elysian gardens of the unknown vector space. No system can be solved without the anchor of the constant.

What, then, makes us think we can ever solve this puzzle, this matrix of people who are nothing but indeterminate? How can we find truth without a single constant in the field?

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The roar of the water beating the rocks into submission is drowned out (quite literally) by the roar of the spectators watching it with hungry eyes and loud mouths. Between the falls and the voices and the splashes of eager feet, nothing else is audible; the forest and the mountain each bow out of the cacophony, counting the measures before they can rejoin the symphony.

As we sit on our rock downstream (watching Nature's majesty spilling out in front of us), I notice a parade of minute soldiers-in-training passing by our ankles. Led by a sergeant oak leaf (the oldest and bravest of the group), the squadron of nature's paraphernalia marches in perfect time with the current. They are mostly aspen leaves, interspersed with a few branches and the occasional gum wrapper, but their path is uniform, their spacing even, each caught in an eddy pushing them away from the falls and down the mountain.

I glance up and notice the general, a little blonde boy hard at work near the edge of the falls. On a rock next to him lie his recruits, willing volunteers he has scooped up from the shore; one by one, he calls them to arms, carefully and meticulously depositing them into the field of battle. And quite the field it is—each tiny recruit must maneuver through a crowd of feet and ankles, a continuously shuffling array of obstacles. It is an intricate dance between order and chaos: the hikers stare upwards, moving to get a better view of the splendor of the falls, yet oblivious to the procession of the leaves and sticks, like clockwork beneath their gaze.

Some of the leafy soldiers meet grisly ends, it is true. But as we sit and gape and talk of majesty, the fringes of creation whirl past us with the water, continuous in their constancy. In front of us, the little blonde boy reaches to grab fistfuls of the water, hurling them with his might towards the sky.

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[1] Whatever that means.
[2] Or infinitely many; consider the counterexample of an underidentified system in Mendelson et al. (1978).

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I find it strange to consider, but there exist people who hate math. I would go so far as to say that the subset of the population which *doesn't* turn up its nose at the intermingling of numbers and letters has measure zero. There are those for whom math is a wholly foreign language that simply refuses to transform itself into sense—the ones who *just don't get it*. Then, there are those for whom math is an annoyance, another hoop to be jumped through in the circus of the modern American education—the ones who *will never use algebra in real life*.

Then, there is my father, who flagrantly and flamboyantly flies in the face of all that is mathematically sound. For him, there is no such thing as foolproof. For him, algebra is a game of tic-tac-toe. When we were children, he went so far as to claim that one times one would equal two, citing the classic "if I have one cookie, and then I give you another cookie, you have two" argument.

Naturally, he raised three math majors. As convincing as the proof-by-cookie argument was,[3] we found his lack of understanding of logic disconcerting. Math, we used to tell him, is not a creed to be adopted, nor a worldview that can be taken on and off like a jacket. There is no room for faith in mathematics—something either *is* or it is *not*. Mathematics and logic are the great arbiters of the universe, the court through which all truth must pass. And whether you liked the result or not did not matter; if you agreed with the axioms, you implicitly adopted the consequences.

* * *

My dog has decided to have a tea party. We clamor and laugh around our island oasis, a boisterous tide of chopping onions and searing salmon ebbing and flowing with the music. Meanwhile, he quietly slinks around each of us, head turned just out of sight. Moving with the liquidity of a Slinky and the silence of a burglar, he disappears around the corner of the couch into no-man's land, deposits something, and returns, darting between legs and down the stairs, only to return moments later and repeat the process.

When eventually I decide to go and investigate, I find a ring of your stuffed animals, arranged in the most perfect circle an Australian Shephard can be expected to create. At the head of the party (facing the rest with a quizzical, almost outraged look) is Rosie, the giraffe. She is flanked by the pterodactyl and the dragon, two bouncers keen on guarding their African queen. And with the regality of Cleopatra and the pose of Mandela, she looks out over her assembly, as if daring me to interrupt her solemn assembly.

There is a huff from behind me. Kingsley has returned, this time with a zebra, and I'm standing in its spot. I retreat from the party, the dog's accusatory glare following me back to the ruckus of the kitchen.

* * *

It took me three years of upper-level math, but in the end, I discovered my father was right.

There is a field of mathematics called analysis,[4] dedicated primarily to the detection of *when* certain types of math (largely integral and derivative calculus) can be performed on functions in different spaces. In broad strokes, it focuses on how these spaces are classified and explored, mainly with the intent of "measuring" different subsets of the space.

In 1924, two mathematicians specializing in analysis proved that given any N-dimensional space, it is possible to take a solid ball and divide it into a finite number of disjoint subsets, which one can then take and reassemble in a different way to create two identical balls of the same volume

[3] And it is categorically true that there exist few things more convincing than cookies.
[4] Which is just about as vague as it sounds; for further examples, see Hungerford (1976).

as the first.[5] Of course, these disjoint subsets are not "pieces" of the ball in the usual sense—it wouldn't be sufficient to chop up a soccer ball by its hexagonal outer arcs and attempt to create two congruent soccer balls from the remainder. Instead, the pieces are an infinite number of points scattered around the ball's interior, making the disassembly relatively infeasible.

However, once these disjoint subsets of points are identified, the reassembly process is simple; no change of shape is necessary to create the two new balls, nor do you even have to adjust the paths of these subsets to avoid collision. You could literally *pull* two identical balls out of the first, each of the same volume and shape as the original.

What's more, the paradox extends further: given a solid ball in an N-dimensional space, it is possible to create a solid ball of *any* other volume in the same space. This means that you could take a soccer ball and rearrange it into a planet, or an electron, or an entire circular universe. All you would need is the capacity to divide the ball into infinitesimally small points.

Essentially, it is possible for one times one to equal two.

* * *

There is nothing more divisive within branches of mathematics than definitions. Math is nothing if not an application of logic, a boat pushing off from the safe harbor of universally accepted axioms to foray into the dark, choppy seas of the unknown Banach space. *Logically*, then, it would follow that the definitions mathematicians use to describe their universe would be the same.

This, however, is not the case. Terms and jargon are appropriated across fields and time and countries, as if all mathematicians were children choosing their favorite word from the finite toy box of available permutations and ignoring the uncountably infinite complexity available to them. Groups, spaces, rings, fields, manifolds—they can be normed without being normal,[6] or be separable with no viable separation existing.

Even within a subfield of mathematics, a term can carry dozens of different connotations. A metric space can be Hausdorff regular to some topologists while to others, the term is superfluous and childishly redundant. There isn't even universal agreement on what constitutes a point in the real plane—is it an ordered set of coordinate indicators, or an infinitesimally small neighborhood of light?

How can such a fragmented study possibly approximate a measure of universal truth? If all mathematicians seek after the proofs found in God's book[7], shouldn't we at least agree on the starting point?

We are as blind men brought before an elephant—a geometer feels the tail and claims that truth is the Euclidean plane, while an algebraic topologist feels the trunk and claims that the universe is more aptly described by a multi-dimensional knot. We reach out, search out, breathe out—but for all this, our hands reel in nothing. We are left with only our preconceived ideas, and we call them real.

* * *

It's one of those days when leaves willingly desert their posts on branches, launching themselves with the gusts into sweeping, Fibonacci-style arcs, only to be interrupted by tiny hands

[5] See Banach and Tarski, "Sur la décomposition des ensembles de points en parties respectivement congruentes," (1924).

[6] And never did a term have such an abnormal meaning.

[7] See Erdös, 1979.

stretched upward, snatching at the warm shades of red and gold. Each time a leaf is caught, the children squeal with laughter, holding their trophy high above the heads of the rest shouting, "I got one! I got one!"

Soon, it becomes a game: children race all over the park, catching as many leaves as they can before they hit the ground. Leaves on the ground are off-limits; they're poison. So are the leaves still clinging resolutely to their branches; to take them off is cheating. And cheaters never prosper.

Some children are whirling dervishes of energy, dashing from tree to tree in an attempt to claim leaves from every tree. Others (on average, the more successful ones) are more patient, circling one tree like a wildcat watching its prey, snatching at every leaf dislodged by each new bluster. Some children favor the red ones, dashing across the yellowing grass with their shirts made into kangaroo pouches to store the glittering rubies. Others prefer the gold leaves, small Kings and Queens Midas with their fingers straining towards the sky. No one bothers with the brown leaves.

I watch from the vantage point of my bench as the dragons amass their treasure troves. Within minutes, though, one girl (presumably dissatisfied with the output of her tree) notices an old brown leaf below her, its edges already curling up with *rigor mortis*. When she picks it up, the curls wrap around her ears, embracing her entire face. She giggles loudly, makes some faces with the leaf, then places it gently back on the ground. After a moment of careful consideration, she stomps on it.

The children nearby hear her shrieks of laughter, and suddenly they, too, are fascinated with the tapestry of decay at their feet. The sound of crunching takes over the park, as small boots leap and pounce on their carefully collected treasures, now unsuspecting victims. Above the children's heads, sparkling gems continue their spiral dives to the ground in peace.

* * *

The impetus of creation lies in its singularity: it is an unsolvable puzzle. In 1931, Kurt Gödel[8] proved that any system of understanding the universe will be fundamentally lacking in at least one aspect. Basic algebra cannot be proven without assuming calculus. Calculus cannot be proven without assuming properties of measure theory and functional analysis, which rest on the assumptions of complex analysis, and so on[9]. No matter how believable an axiom or how thorough a model construction, something will be missing. The universe simply contains too many variables.

Why then, dare we exhaust every medium to express the uniqueness and *complexity* of the human individual, while in the same breath turning to the much grander universe to demand a simple, holistic answer that would fit on a scrap of paper jammed into our pockets? The modern society puts everything on a continuum, from political preferences to gender, while religions claim a monopoly on the good[10] and physicists are brash enough to search for a theory of *everything*. We want to be special, and yet we want the world around us to fit into our unyielding schema.

There are unmeasurable sets whose elements will never be known and unique solutions to differential equations that we will never uncover. There will always be Pandora's boxes we can never open, while in other ways, the cat will *always* be both alive and dead.

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[8] See K. Gödel, "Ueber formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I" *Monatsh. Math. Physik*, (1931).

[9] Turtles all the way down, as it were (see Hawking, 1988).

[10] At best, an arbitrary term.